Journal of Sound and Vibration (1998) **209**(3), 519–530

ON THE CONTRIBUTION FROM SKIN STEPS TO BOUNDARY-LAYER GENERATED INTERIOR NOISE

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(*Received* 26 *February* 1997, *and in final form* 2 *September* 1997)

An estimate is made of the aircraft interior noise produced by high subsonic turbulent wall pressures interacting with a fuselage skin step formed when adjacent, elastic panels overlap. The panels are modelled as thin elastic plates bonded in the vicinity of the step, and clamped along a line transverse to the mean flow direction. Sound is produced by scattering of the turbulence induced flexural skin motions at the clamp line, and by interaction of the turbulence pressure with the step. The skin step component is found to be significant at high frequencies, *above* the convective resonance frequency of the panels (at which the turbulence convection velocity equals the flexural wave speed). At typical cruise Mach numbers, however, when boundary layer generated sound is believed to dominate interior cabin noise, the critical frequency above which skin step noise is important is of the order of 10 kHz, which is beyond the range of interest in practice. The critical frequency is much lower at take-off and landing approach conditions, but the turbulent boundary layer is then a relatively unimportant contributor to cabin noise.

1. INTRODUCTION

High speed turbulent flow over an aircraft fuselage is responsible for a substantial component of the interior noise [1–4], and is probably the most important source of cabin noise for jet powered passenger aircraft in steady cruise. The principal mechanism of sound generation in subsonic flight is the interaction of boundary layer pressures with structural inhomogeneities, such as ring-stiffeners, stringers and windows, which produce sound whose amplitude and frequency content typically vary widely with flight speed, even though the boundary layer characteristics are believed to be only weakly dependent on flow velocity, at least within the subsonic regime.

Several numerical schemes are being developed to predict the boundary layer generated interior noise [5]. Graham [6, 7] has proposed a semi-analytical model that is applicable to turbulent flow over an isolated fuselage panel, and which incorporates important structural effects of interior trim. None of these methods explicitly includes the influence of the exterior mean flow in calculating the noise. A simplified treatment that involves mean flow was proposed by Howe and Shah [8], who considered the interaction of a boundary layer with a nominally smooth elastic plate with periodically spaced rib-stiffeners. An elementary model of "coincidence scattering", i.e., of sound generation by the edge scattering of flow excited structural waves whose phase velocity exactly matches the convection velocity of a flow inhomogeneity, was discussed in reference [9], but no account was taken of mean flow. However, although there is currently no fully satisfactory prediction scheme, the basic principles appear to be well understood.

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Most numerical prediction procedures take some account of lateral curvature of aircraft panels (e.g., see references [5, 10]) and of interior panel trim, but there is no quantified understanding of the influence on sound generation of the step-like discontinuities in the outer skin that occur where adjacent fuselage panels overlap. This might be important when the step height is comparable to the boundary layer displacement thickness, so that the interaction between the turbulence and step is likely to be strong. Sound is generated by the scattering of turbulence pressures by a step, but in addition, the boundary layer wall pressures downstream of the step are also modified, and this will in turn affect the production of sound by direct panel forcing (e.g., see references [11, 12]).

In this paper an analysis is performed to estimate the relative contribution from skin steps to boundary layer interior noise. The method is a development of that used by Howe and Shah [8] for a ribbed panel, which explicitly incorporates the influence of the finite Mach number of the boundary layer. No account is taken in this calculation of the changes in boundary layer structure caused by the step, which is therefore assumed to generate sound by interaction with a pre-existing turbulent field that is swept over the step, as in the analogous problem discussed in reference [13] for a step in a *rigid* wall. Such changes are associated with separation induced at the step and will be important when the step height is comparable to the boundary layer displacement thickness. From an acoustic point of view separation tends to *reduce* the radiated sound relative to that predicted when the unsteady motion is modelled according to the proposed inviscid scattering theory [14, 15], because "noisy" potential flow singularities are smoothed-out. The principal conclusion of this investigation is that, for typical aircraft structures and cruise Mach numbers, the influence of skin steps on interior noise is small except at very high frequencies, which are probably too high to be relevant in practice.

The analytical model is formulated in section 2, where the sound generated by the interaction of the turbulent flow with a skin joint is expressed in terms of the unsteady, boundary layer wall pressure and a suitable acoustic Green's function that includes the combined effects of structural compliance, the skin step and a rib stiffener at the panel overlap. This is used (in sections 3, 4) to investigate the dependence on Mach number of skin step noise relative to the noise generated by interaction of the boundary layer with the clamp line of adjacent panels.

2. THE AERODYNAMIC SOUND PROBLEM

2.1. FORMULATION

Consider high subsonic mean flow over the idealized skin step depicted schematically in Figure 1(a), which consists of two skin sections, modelled as semi-infinite, thin elastic overlapping plates which are clamped together along a nominally rigid, rectilinear rib or stringer. The exterior fluid has mean density ρ_1 and sound speed c_1 with mean flow in the x_1 direction of the rectangular coordinate system (x_1, x_2, x_3), as indicated in the figure. The overlapping skin sections are in practice riveted along the clamp line, which is taken to coincide with the x_3-axis.

When the influence of the skin step on sound production is ignored, the skin thickness h, say, is taken to be sufficiently small that both plates may be assumed to lie in the plane $x_2 = 0$, where $x_2 > 0$ in the exterior mean flow. This flow is assumed to be turbulent, and the problem of calculating the sound produced by the interaction of the turbulence with the plates in the neighborhood of the clamp line was considered in reference [8]. The basis of this method will be briefly recalled, and the extension necessary to take account of the skin step will then be described.

The fluid in the "interior" domain $x_2 < 0$ has mean density and sound speed equal to ρ_0 and c_0, respectively, and is in a mean state of rest. To determine the sound radiated into this region, the configuration of Figure 1(a) is first taken in the simplified form illustrated in Figure 1(b), where the skin step is ignored, and the adjacent skin sections are regarded as smoothly clamped together along the x_3 -axis (at O). If U denotes the mean flow speed (in the x_1 direction) in the exterior region outside the turbulent boundary layer, where the Mach number $M = U/c_1 < 1$, the equations describing the production of aerodynamic sound can be taken in the form

$$[(1/c_o^2)\,(\partial^2/\partial t^2) - \nabla^2]\,\mathscr{B} = 0, \qquad x_2 < 0, \tag{1}$$

$$[(1/c_1^2)\,(\partial/\partial t + U\partial/\partial x_1)^2 - \nabla^2]\,\mathscr{B} = Q(x,\,t), \qquad x_2 > 0, \tag{2}$$

where $\mathscr{B} = w + \frac{1}{2}v^2$ is the total enthalpy, w being the fluid specific enthalpy, $\mathbf{v}$ the velocity, and t denotes time [16]. The source term $Q(\mathbf{x},\,t)$ is non-zero only within the boundary layer, and includes all effects associated with the boundary layer mean shear flow and the turbulence sources.

When the motion is isentropic, the acoustic pressure fluctuation p can be obtained from the solution of equations (1) and (2) by means of the relation $\partial p/\partial t = \rho \mathrm{D}\mathscr{B}/\mathrm{D}t$, where ρ is the local mean density and $\mathrm{D}/\mathrm{D}t$ is the material derivative. In those regions where the flow is irrotational, $\mathscr{B} \equiv -\partial\phi/\partial t$, where $\phi(\mathbf{x},\,t)$ is the velocity potential of the unsteady motion.

Deflections of the plates are assumed to be governed by the linearized thin plate equation

$$[D(\partial^2/\partial x_1^2 + \partial^2/\partial x_3^2)^2 + m\,\partial^2/\partial t^2]\zeta + [p] = 0, \tag{3}$$

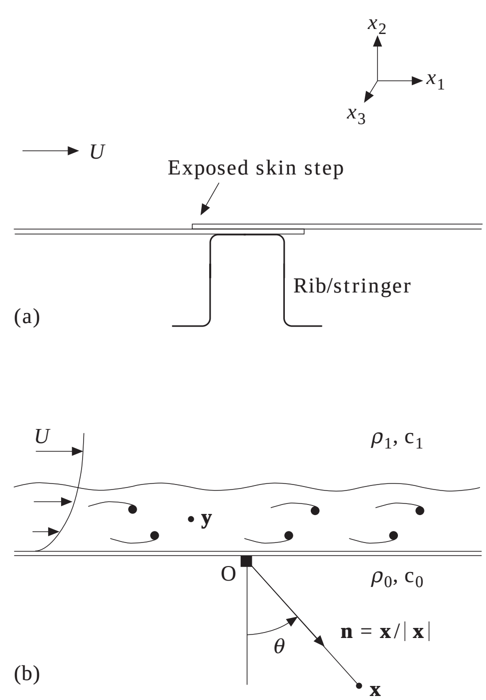

Figure 1. (a) Schematic skin-step configuration. (b) Simplified model in absence of skin step.

where $\zeta(x_1, x_3, t)$ is the flexural displacement in the x_2 direction, D the plate bending stiffness, m the mass per unit area, and

$$[p] = p(x_1, +0, x_3, t) - p(x_1, -0, x_3, t)$$

is the pressure loading. The displacement ζ and the pressure are also related by the linearized kinematic relations

$$\frac{\partial^2 \zeta}{\partial t^2} = \frac{-1}{\rho_o}\frac{\partial p}{\partial x_2}, \qquad x_2 = -0; \qquad \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)^2 \zeta = \frac{-1}{\rho_1}\frac{\partial p}{\partial x_2}; \qquad x_2 = +0. \tag{4}$$

Along the clamp line the displacement satisfies the clamped edge conditions

$$\zeta = 0, \qquad \frac{\partial \zeta}{\partial x_1} = 0, \qquad x_3 = 0. \tag{5}$$

2.2. GREEN'S FUNCTION IN THE ABSENCE OF THE SKIN STEP

Green's function $G_o(\mathbf{x}, \mathbf{y}, t-\tau)$ for the coupled equations (1) and (2) is the solution with outgoing wave behavior, which also satisfies the appropriate conditions on the plates and so on the clamp line, when Q on the right of equation (2) is replaced by the point source $\delta(\mathbf{x}-\mathbf{y})\delta(t-\tau)$ concentrated at $\mathbf{y}$. When G is known the acoustic pressure in the interior region (where $\mathscr{B} \approx p/\rho_o$) can be written

$$p(\mathbf{x}, t) = \rho_o \int G_o(\mathbf{x}, \mathbf{y}, t-\tau) Q(\mathbf{y}, \tau)\, \mathrm{d}^3\mathbf{y}\, \mathrm{d}\tau, \tag{6}$$

where the spatial integration is over the boundary layer source region and the time integration is over $-\infty < \tau < \infty$.

It will suffice for the present discussion to confine attention to the generation of sound by boundary layer disturbances $Q \equiv Q(x_1, x_2, t)$ that are independent of the spanwise co-ordinate x_3. This is a special case of the fully three-dimensional analysis given in reference [8], and is adequate for estimating the relative contribution to the sound from the skin step. For an observer at $\mathbf{x}$ in the interior domain, and at large distances from the clamp line, one then has (see reference [8] for details)

$$G_o(\mathbf{x}, \mathbf{y}, t-\tau) \approx \frac{\sqrt{\mathrm{i}\rho_1/\rho_o}}{2\pi(8\pi|\mathbf{x}|)^{1/2}} \int_{\infty}^{\infty} \frac{T(\omega, \theta)}{\sqrt{k_o}} \left(\mathrm{e}^{-\mathrm{i}k_o(n_1 y_1 - N_2 y_2)} + \frac{N_2}{c_1(c_o/c_1 - M\sin\theta)} \right.$$

$$\left. \times \sum_{n=0}^{3} \int_{-\infty}^{3} \frac{(\omega + Uk)a_n k^n \mathrm{e}^{\mathrm{i}(ky_1 + \Gamma_+(k)y_2)}\,\mathrm{d}k}{\Gamma_+(k)\mathscr{L}_+(k, \omega)} \right) \mathrm{e}^{-\mathrm{i}\omega(t-\tau-|\mathbf{x}|/c_o)}\,\mathrm{d}\omega, \quad |\mathbf{x}| \to \infty, \tag{7}$$

where the integration contour in the k-plane is indented to pass respectively above and below real poles and branch points of the integrand in $k \lessgtr 0$; $T(\omega, \theta)$ is the elastic plate, plane wave transmisson coefficient

$$T(\omega, \theta) = \frac{-2\mathrm{i}\varepsilon(c_1/c_o)(c_o/c_1 - M\sin\theta)}{N_2\left\{\left[\left(\frac{\omega}{\omega_c}\right)^2 \sin^4\theta - 1\right]\frac{\omega}{\omega_c} - \mathrm{i}\varepsilon\left[\frac{1}{\cos\theta} + \frac{\rho_1 c_1^2/\rho_o c_0^2}{N_2}\left(\frac{c_o}{c_1} - M\sin\theta\right)^2\right]\right\}}, \tag{8}$$

and the remaining new terms appearing in these formulae are defined as

$$n_1 = \sin\theta, \qquad N_2 = \sqrt{(c_o/c_1 - M\sin\theta)^2 - \sin^2\theta}, \qquad \varepsilon = \rho_o c_o/\omega_c m, \qquad \omega_c = c_0^2 (m/D)^{1/2},$$

θ being the angle shown in Figure 1(b) defining the observer direction,

$$\gamma(k) = (\kappa_o^2 - k^2)^{1/2}, \qquad \Gamma_\pm(k) = [(\omega/c_1 \pm Mk)^2 - k^2]^{1/2}, \qquad \kappa_0 = \omega/c_0,$$
$$\mathscr{L}_\pm(k,\omega) = Dk^4 - m\omega^2 - \mathrm{i}[\rho_o\omega^2/\gamma(k) + \rho_1(\omega \pm Uk)^2/\Gamma_\pm(k)], \tag{9}$$

where branch cuts in the upper and lower halves of the complex k-plane are taken such that $\gamma(k)$ and $\Gamma_\pm(k)$ have positive imaginary parts on the real axis. Here, $\omega_c > 0$ is the *coincidence frequency* above which the *in vacuo* phase speed of flexural waves on the plate exceeds the interior sound speed c_o. The coefficient ε is a measure of the influence of fluid loading on the motions of the plates. For an aluminum plate of thickness h in air, $\varepsilon \approx 0{\cdot}0022$ and $\omega_c h/c_o \approx 0{\cdot}22$. The four coefficients a_n are complex functions of the frequency ω, whose values depend on conditions at the junction of two plates, and satisfy $a_n(-\omega) = a_n^*(\omega)$, where the asterisk denotes complex conjugate. For the clamped condition considered here [8],

$$a_0 = (m\omega^2/K_o)A_0, \qquad a_1 = (m\omega^2/K_o^2)A_1, \qquad a_2 = 0, \qquad a_3 = 0, \tag{10}$$

where $K_0 = (m\omega^2/D)^{1/4} > 0$ is the *in vacuo* wavenumber of bending waves of frequency ω, and the dimensionless coefficients A_0, A_1 are given by

$$A_0 = \frac{\Psi_2 + \mu n_1 \Psi_1}{\Psi_1^2 - \Psi_0\Psi_2}, \qquad A_1 = \frac{-(\Psi_1 + \mu n_1 \Psi_0)}{\Psi_1^2 - \Psi_0\Psi_2}, \qquad \mu = \sqrt{\omega/\omega_c}, \tag{11}$$

$$\Psi_j = \int_{-\infty}^{\infty} \frac{\lambda^j \,\mathrm{d}\lambda}{\left[\lambda^4 - 1 - \dfrac{\mathrm{i}\varepsilon}{\mu}\left(\dfrac{1}{\sqrt{\mu^2 - \lambda^2}} + \dfrac{(\rho_o c_o^2/\rho_1 c_1^2)(c_o/c_1 + M\lambda/\mu)^2}{\sqrt{(\mu c_o/c_1 + M\lambda)^2 - \lambda^2}}\right)\right]}. \tag{12}$$

The path of integration along the real axis is indented to pass respectively above and below poles and branch points of the integrand in $\lambda \lessgtr 0$.

2.3. SKIN STEP MODIFIED GREEN'S FUNCTION

In the presence of the skin step the representation (6) becomes

$$p(\mathbf{x},t) = \rho_o \int \{G_o(\mathbf{x},\mathbf{y},t-\tau) + G_s(\mathbf{x},\mathbf{y},t-\tau)\} Q(\mathbf{y},\tau)\,\mathrm{d}^3\mathbf{y}\,\mathrm{d}\tau, \tag{13}$$

where G_s is the correction to Green's function (7) produced by the presence of the step. To calculate G_s one first recalls that the reciprocal theorem [8, 17, 18] implies that the term in the large brackets of the integrand of equation (7), regarded as a function of $\mathbf{y}$, is proportional to the velocity potential in $y_2 > 0$ generated by a point source at the observer location $\mathbf{x}$ when the mean flow in the exterior region is *reversed* in direction. This potential must satisfy boundary conditions of the type (3–5) in the reverse flow problem. Thus, if one assumes the step to be in the immediate neighborhood of the clamp line $y_3 = 0$ (as in Figure 2), the presence of the step implies that the normal surface derivative $\partial G/\partial y_n \equiv \partial(G_o + G_s)/\partial y_n$ should vanish on the step profile in the reversed flow problem. In the reciprocal problem the length scale of the unsteady fluid motion in the neighborhood of the step *when the step is ignored* is of order $1/K_o \leqslant 1/\kappa_o$. If $K_o h \leqslant 1$, i.e., if the structural wavelength is large compared to the skin thickness, which is usually the case (and is

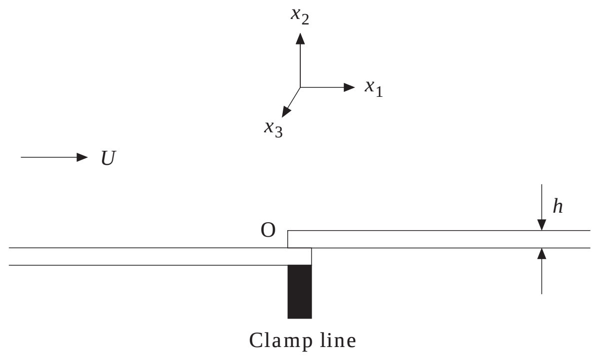

Figure 2. Idealization of skin step and clamp line.

actually necessary to justify the use of the thin plate equation (3)), the velocity potential of the unsteady motion in the neighborhood of the step *relative to that of the plates* is equal to a sum of terms proportional respectively to $n_1[y_1 + \phi^*(\mathbf{y})]$ and $k[y_1 + \phi^*(\mathbf{y})]$, where $\phi^*(\mathbf{y})$ is a solution of the homogeneous, time harmonic form of equation (2) when the mean flow is *reversed*, which satisfies

$$\partial[y_1 + \phi^*(\mathbf{y})]/\partial y_n = 0 \qquad \text{on the step profile.} \tag{14}$$

This is because, near the step, the exponentials in equation (7) can be expanded to first order in $\mathbf{y}$; the term y_1 must then be augmented by ϕ^* in order to satisfy equation (14). If the co-ordinate origin is taken at the foot of the step, then to first order in the step height s (which is equal to plate thickness h for simple overlapping plates), the boundary condition (14) becomes

$$\partial\phi^*/\partial y_2 = \pm s\delta(y_1) \quad \text{on} \quad y_2 = +0,$$

and the appropriate ϕ^* is accordingly found to be given by

$$\phi^*(\mathbf{y}) = \frac{\pm s}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{\mathrm{i}(ky_1 + \Gamma_+(k)y_2)}}{\Gamma_+(k)}\,\mathrm{d}k, \tag{15}$$

where the $\pm$ sign is taken according as the step is forward or backward facing relative to the mean flow. One then has

$$G_s(\mathbf{x}, \mathbf{y}, t-\tau) \approx \frac{(\rho_1/\rho_o)\phi^*(\mathbf{y})}{2\pi\sqrt{8\pi \mathrm{i}|\mathbf{x}|}} \int_{-\infty}^{\infty} \frac{T(\omega, \theta)}{\sqrt{\kappa_o}} \left\{ \kappa_o n_1 - \frac{N_2}{c_1(c_o/c_1 - M\sin\theta)} \right.$$

$$\left. \times \sum_{n=0}^{3} \int_{-\infty}^{\infty} \frac{(\omega + Uk)a_n k^{n+1}\,\mathrm{d}k}{\Gamma_+(k)\mathscr{L}_+(k,\omega)} \right\} \mathrm{e}^{-\mathrm{i}\omega(t-\tau-|x|/c_o)}\,\mathrm{d}\omega. \tag{16}$$

The integral within the brace brackets converges only for $n \leqslant 1$; in particular it converges for the clamped condition, for which $a_2 = a_3 \equiv 0$. This is because when $a_2 \neq 0$ there is a discontinuity in the slope of the two plates at the join, and there can then be no smooth flow over the plates in the absence of the step (see reference [8]), as assumed in the reciprocal problem.

3. THE INTERIOR ACOUSTIC PRESSURE

3.1. THE BLOCKED WALL PRESSURE

It is convenient to express the interior radiation in terms of the boundary layer wall pressure on the exterior skin that would be generated when the wall is regarded as rigid. This is the *blocked* pressure, which can be calculated by solving equation (2) subject to the boundary condition $\partial\mathscr{B}/\partial x_2 = 0$ on $x_2 = +0$, and usng the relation $\partial p/\partial t \approx \rho_1 (\partial/\partial t + U\partial/\partial x_1)\mathscr{B}$. If $p_b(k_1, \omega)\, \mathrm{e}^{\mathrm{i}(k_1 x_1 - \omega t)}$ is the component of the blocked pressure of frequency ω and wavenumber k_1 in the streamwise direction, then it is readily shown that

$$p_b(k_1, \omega) = [2\pi \mathrm{i}\rho_1 (\omega - Uk_1)/\omega\Gamma_-(k_1)]\, \hat{Q}(k_1, -\Gamma_-(k_1), \omega), \tag{17}$$

where

$$\hat{Q}(k_1, k_2, \omega) = \frac{1}{(2\pi)^3}\int\int_{-\infty}^{\infty} \mathrm{d}x_1\, \mathrm{d}t \int_0^{\infty} Q(x_1, x_2, t)\, \mathrm{e}^{-\mathrm{i}(k_1 x_1 + k_2 x_2 - \omega t)}\, \mathrm{d}x_2. \tag{18}$$

The dominant components of the blocked wall pressure occur in the vicinity of $k_1 = \omega/U_c$ where U_c is a convection velocity approximately equal to $0{\cdot}7U$ [19–22].

3.2. SOUND RADIATED FROM THE CLAMP LINE

The acoustic pressure $p_o(\mathbf{x}, t)$, say, produced by the interaction of the boundary layer pressure field with the clamp line is given by equation (6). The contribution from the first, exponential term in the round brackets in the representation (7) of G_o can be ignored, since it does not correspond to the generation of sound at the clamp line, but to the transmission through the wall of boundary layer pressure fluctuations that are *already* sound, exactly as if the clamp were absent. Relations (17) and (18) can be used to express the contribution from the second term in the round brackets of (7) in the form

$$p_o(\mathbf{x}, t) \approx \sqrt{\frac{\pi}{2i|\mathbf{x}|}}\frac{N_2}{(c_1/c_o)(c_o/c_1 - M\sin\theta)}$$

$$\times \sum_{n=0}^{3}\int_{-\infty}^{\infty} \frac{T(\omega, \theta)\sqrt{\kappa_o}\, a_n(-k_1)^n}{\mathscr{L}_-(k_1, \omega)}\, p_b(k_1, \omega)\, \mathrm{e}^{-\mathrm{i}\omega(t - |\mathbf{x}|/c_o)}\, \mathrm{d}k_1\, \mathrm{d}\omega. \tag{19}$$

3.3. SOUND RADIATED FROM THE SKIN STEP

Similarly, the contribution $p_s(\mathbf{x}, t)$ to the acoustic pressure from the step, given by the term in G_s in equation (13) becomes

$$p_s(\mathbf{x}, t) \approx \frac{\mp s}{2\pi}\sqrt{\frac{\pi}{2\mathrm{i}|\mathbf{x}|}}\int\int_{-\infty}^{\infty} \frac{\sqrt{\kappa_o}\,\omega T(\omega, \theta)}{(\omega - Uk_1)}\Bigg\{\sin\theta - \frac{N_2}{\kappa_o c_1 (c_o/c_1 - M\sin\theta)}$$

$$\times \sum_{n=0}^{3}\int_{-\infty}^{\infty} \frac{(\omega + Uk)a_n k^{n+1}\, \mathrm{d}k}{\Gamma_+(k)\mathscr{L}_+(k, \omega)}\Bigg\}\, p_b(k_1, \omega)\, \mathrm{e}^{-\mathrm{i}\omega(t - |\mathbf{x}|/c_o)}\, \mathrm{d}k_1\, \mathrm{d}\omega, \tag{20}$$

the upper/lower sign being taken respectively for forward and backward facing steps.

3.4. ACOUSTIC EFFICIENCIES

Consider the sound generated by the interaction of the component $p_I \equiv p_b(k_1, \omega)\,\mathrm{d}k_1\,\mathrm{d}\omega$ of the blocked wall pressure with the clamp line and skin step. The relative efficiencies of sound generation by these two mechanisms is determined by the relative magnitudes of the ratios $p_o(\mathbf{x}, t)/p_I$ and $p_s(\mathbf{x}, t)/p_I$. Let one suppose that p_I is characteristic of the dominant wall pressure fluctuations, for which $\omega/k_1 = U_c$. It then follows from equations (19) and (20), using also results given in section 2, that

$$\frac{p_o}{p_I} \approx \sqrt{\frac{\pi h}{2i|x|}} \times \frac{N_2\,T(\omega, \theta)\mu^2 \bar{M}_c^4\,[A_0 - \mu A_1/\bar{M}_c]\,\mathrm{e}^{-\mathrm{i}\omega(t-|\mathbf{x}|/c_o)}}{\sqrt{\frac{\omega_c h}{c_o}\frac{c_1}{c_o}\left(\frac{c_o}{c_1} - M\sin\theta\right)}\left\{(\mu^4 - \bar{M}_c^4)\mu^2 - \varepsilon \bar{M}_c^5\left[\frac{1}{\sqrt{1-\bar{M}_c^2}} + \frac{(\rho_1/\rho_o)(1 - M/M_c)^2}{\sqrt{1-(M-M_c)^2}}\right]\right\}}, \tag{21}$$

$$\frac{p_s}{p_I} \approx \pm\sqrt{\frac{\pi h}{2\mathrm{i}|\mathbf{x}|}}\,\frac{(s/h)\sqrt{(\omega_c h/c_o)}\,\mu T(\omega, \theta)}{2\pi(M/M_c - 1)}\left[\sin\theta - \frac{N_2\,[A_0\,\Phi_0 + A_1\,\Phi_1]}{c_o/c_1 - M\sin\theta}\right]\mathrm{e}^{-\mathrm{i}\omega(t-|\mathbf{x}|/c_o}, \tag{22}$$

where

$$M_c = U_c/c_1, \qquad \bar{M}_c = U_c/c_0,$$

and

$$\Phi_n = \int_{-\infty}^{\infty} \frac{(c_o/c_1 + M\lambda/\mu)\lambda^{n+1}\,\mathrm{d}\lambda}{\sqrt{\left(\frac{\mu c_o}{c_1} + M\lambda\right)^2 - \lambda^2}\left\{\lambda^4 - 1 - \frac{\mathrm{i}\varepsilon}{\mu}\left[\frac{1}{\sqrt{\mu^2 - \lambda^2}} + \frac{(\rho_1 c_1^2/\rho_o c_o^2)(c_o/c_1\,\mu + M\lambda)^2}{\mu^2\sqrt{(\mu c_o/c_1) + M\lambda)^2 - \lambda^2}}\right]\right\}}. \tag{23}$$

4. NUMERICAL RESULTS

The curves in Figures 3–5 are plots of the calculated efficiency 20 log $(|p(\mathbf{x}, t)/p_I|/\sqrt{h/|\mathbf{x}|})$ (dB) of interior sound production by the clamp line and by the skin step at three different mean flow Mach numbers M, and for the nominal radiation direction $\theta = 45^\circ$ (see Figure 1(b)). The calculations have been performed for aluminum plates in air; for simplicity it is assumed that $\rho_o = \rho_1$, $c_o = c_1$, for which the remaining non-dimensional parameters describing the fluid-structure interaction have been assigned the values $\varepsilon = 0{\cdot}0022$, $\omega_c h/c_o = 0{\cdot}22$, $M_c = 0{\cdot}7M$.

The peaks labeled "convective resonance" occur where the convection velocity $\omega/k_1 = U_c$ of the blocked wall pressure component $p_b(k_1, \omega)$ is the same as the phase speed of bending waves in the plates of the same frequency and wavenumber, and occurs approximately at $\omega = M_c^2\,\omega_c$. At frequencies exceeding the convective resonance frequency the efficiency of sound generation by scattering at the clamp line decreases very rapidly. At frequencies below resonance the overall level of the skin step generated sound is always negligible. At higher frequencies, however, it becomes dominant because of the precipitous

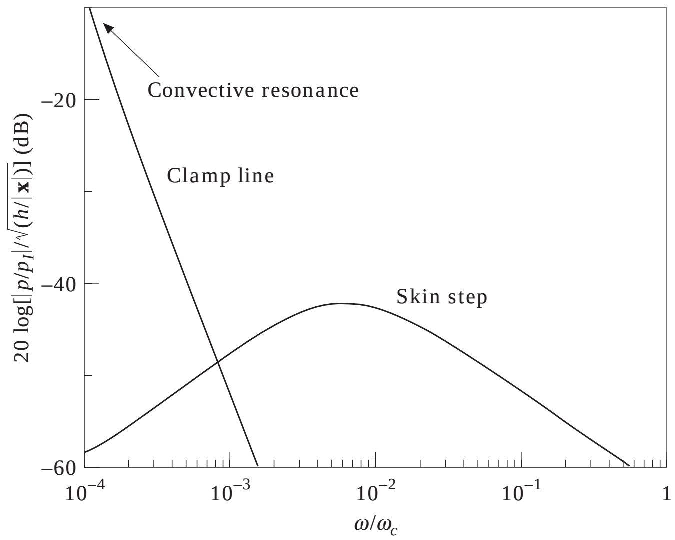

Figure 3. Efficiencies of sound generation by scattering from the clamp line and from the skin step; aluminum skin, $M = 0{\cdot}01$, $\theta = 45^\circ$.

decrease in the efficiency of sound generation by clamp line scattering. The peaks in the skin step sound evident in Figures 4 and 5 occur at sufficiently large frequencies that $\omega/\omega_c = 1/\sin^2\theta > 1$. This condition is satisfied by those structural waves generated at the step whose phase velocity is just equal to the trace velocity of sound waves launched in the observer direction θ, and corresponds to the vanishing of the first term in the brace brackets of the denominator of the definition (8) of the plane wave transmission coefficient $T(\omega, \theta)$.

The figures indicate that the skin step sound will be dominant at sufficiently high frequency. To illustrate the orders of magnitude involved, Table 1 lists approximate values

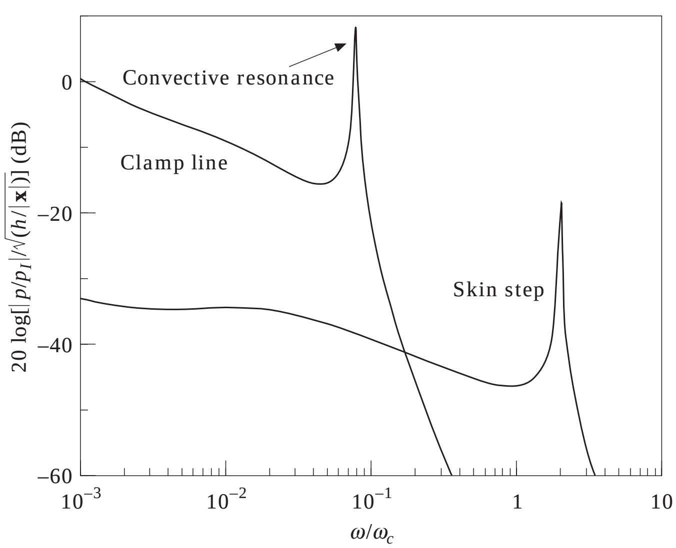

Figure 4. Efficiencies of sound generation by scattering from the clamp line and from the skin step; aluminum skin, $M = 0{\cdot}4$, $\theta = 45^\circ$.

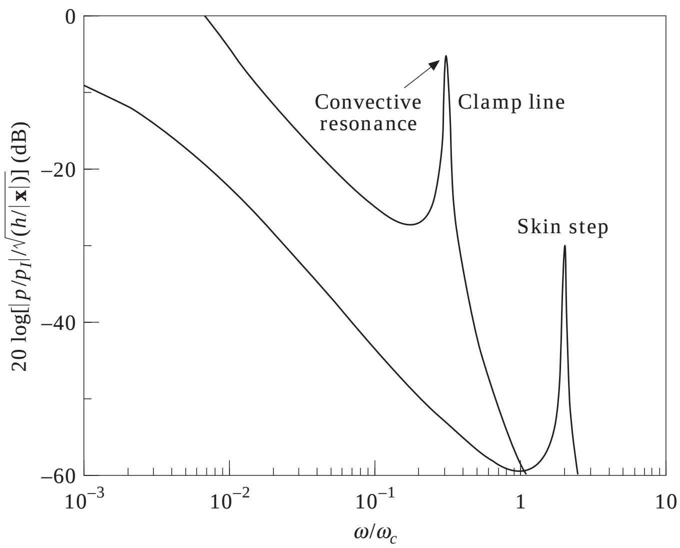

Figure 5. Efficiencies of sound generation by scattering from the clamp line and from the skin step; aluminum skin, $M = 0{\cdot}8$, $\theta = 45^\circ$.

of critical frequency $\omega_s/2\pi \equiv f_s$ Hz, say, above which skin step noise is dominant for the three Mach numbers considered in Figures 3–5 when $\theta = 45^\circ$, and when the skin thickness $h = 0{\cdot}1$ cm.

Thus, at $M \approx 0{\cdot}8$, which is appropriate for cruise conditions at which boundary layer generated interior noise is believed to be significant, it appears that the contribution from skin steps will be unimportant at frequencies less, than, say, 10 kHz. According to Table 1, the critical frequency decreases rapidly with decreasing flight speed, being as small as 2 kHz when $M = 0{\cdot}4$. However, such flight speeds are characteristic of takeoff and landing, when boundary layer generated noise within the cabin is negligible compared to that generated by other sources. The critical frequency varies relatively little with radiation direction θ. This is evident from Figure 6, which depicts the variation of $20 \times \log|p_o/p_s|$ (dB) with frequency for $\theta = -45^\circ, 0^\circ, 45^\circ$.

5. CONCLUSION

Turbulent boundary layer generated noise is one of the dominant components of passenger aircraft interior noise at high subsonic cruise conditions. This noise is produced by the interaction of turbulent pressures with fuselage inhomogeneities, in particular with skin panel junctions. In this paper an analysis has been made to estimate the likely contribution from interaction of the turbulent flow with skin steps, where adjacent panels

TABLE 1

Critical frequency when $\theta = 45^\circ$

Mach number	ω_s/ω_c	f_s Hz
0·01	0·0008	10
0·4	0·17	2000
0·8	1	12000

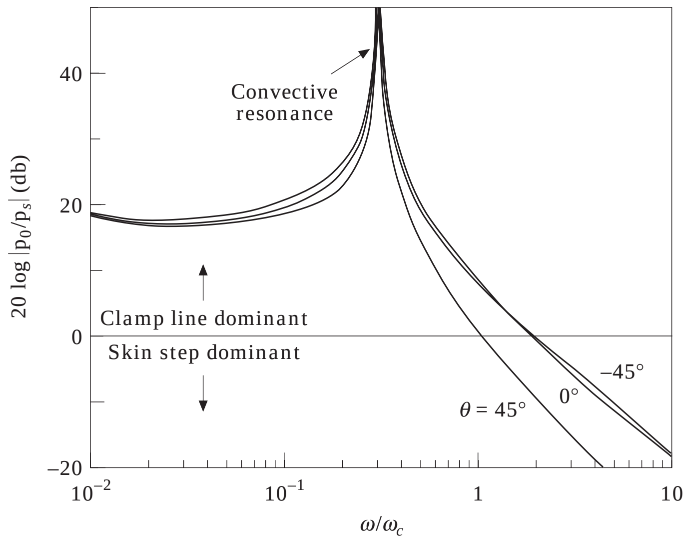

Figure 6. Ratio $|p_o/p_s|^2$ (in dB) of the sound generated respectively by scattering at the clamp line and skin step for different radiation angles θ; aluminum skin, $M = 0{\cdot}8$.

overlap. Sound and structural vibrations are excited by the interaction. In addition, the step causes a localized, temporary change in the boundary layer turbulence characteristics. This has been ignored, and the turbulence has been regarded as *frozen* during its interaction with the step. Skin step noise is found to be significant only at high frequencies, *beyond* the structural convective resonance frequency (at which the boundary layer eddy convection velocity coincides with the phase speed of flexural waves in the skin); above this frequency the efficiency of sound production by the scattering of turbulence pressures at a clamped panel edge decreases precipitously. However, at cruise Mach numbers, where boundary layer generated noise is important, the skin step noise is dominant only at frequencies typically in excess of about 10 kHz, which is beyond the range of interest in practice. At lower flight speeds, appropriate to take-off and landing approach conditions, the skin step contribution is significant at lower frequencies, but in these circumstances cabin interior noise is no longer controlled by the fuselage boundary layer.

ACKNOWLEDGMENT

The work reported here is supported under Grant NAG1-1688 from NASA Langley Research Center, administered by Dr. Richard J. Silcox.

REFERENCES

1. J. F. Wilby, C. D. McDaniel and E. G. Wilby 1985 *NASA* CR-178004. In-flight acoustic measurements on a light twin-engined turboprop airplane.
2. J. S. Mixson and J. F. Wilby 1991 in *Aeroacoustics of Flight Vehicles: Theory and Practice*. **2**, NASA Ref. Pub. No. 1258: Chapter 16; Interior noise.
3. W. R. Graham 1993 Boundary layer noise and vibration. *Doctoral Thesis*, Cambridge University Engineering Department.
4. J. F. Wilby 1996 *Journal of Sound and Vibration* **190**, 545–564. Aircraft interior noise.
5. R. J. Silcox 1995 (editor) *Proceedings of the Interior Noise Workshop*, held at NASA Langley Research Center, 25–27 April.

6. W. R. Graham 1996 *Journal of Sound and Vibration* **192**, 101–120. Boundary layer induced noise in aircraft, Part I: the flat plate model.
7. W. R. Graham 1996 *Journal of Sound and Vibration* **192**, 121–138. Boundary layer induced noise in aircraft, Part II: the trimmed flat plate model.
8. M. S. Howe and P. L. Shah 1996 *Journal of the Acoustical Society of America* **99**, 3401–3411. Influence of mean flow on boundary layer generated interior noise.
9. P. L. Shah and M. S. Howe 1996 *Journal of Sound and Vibration* **197**, 103–115. Sound generated by a vortex interacting with a rib-stiffened elastic plate.
10. W. R. Graham 1995 *Journal of the Acoustical Society of America* **98**, 1581–1595. The influence of curvature on the sound radiated by vibrating panels.
11. T. M. Farabee and M. J. Casarella 1988 *Acoustic Phenomena and Interaction in Shear Flows over Compliant and Vibrating Surfaces—Volume* 6 (Editors: W. L. Keith, E. M. Uram and A. J. Kalinowski.) ASME paper entitled: Wall pressure fluctuations beneath a disturbed turbulent boundary layer, pp 121–135.
12. J. Sulc, J. Hofr and L. Benda 1982 *Journal of Sound and Vibration* **84**, 105–120. Exterior noise on the fuselage of light propeller driven aircraft in flight.
13. M. S. Howe 1989 *Journal of Fluids and Structures* **83**, 83–96. Sound produced by turbulent boundary layer flow over a finite region of wall roughness, and over a forward facing step.
14. M. S. Howe 1976 *Journal of Fluid Mechanics* **76**, 711–740. The influence of vortex shedding on the generation of sound by convected turbulence.
15. M. S. Howe 1988 *Proceedings of the Royal Society* **A420**, 157–182. Contributions to the theory of sound production by vortex–airfoil interaction, with application to vortices with finite axial velocity defect.
16. M. S. Howe 1975 *Journal of Fluid Mechanics* **71**, 625–673. Contributions to the theory of aerodynamic sound, with application to excess jet noise and the theory of the flute.
17. M. S. Howe 1975 *Journal of Fluid Mechanics* **67**, 579–610. The generation of sound by aerodynamic sources in an inhomogeneous steady flow.
18. W. Möhring 1979 *Proceedings of the Symposium on Mechanics of Sound Generation in Flows, Göttingen, August* 28–31 (editor E.-A. Müller: Springer, Berlin). Modelling low Mach number noise, 85–96.
19. B. M. Efimtsov 1982 *Soviet Physics Acoustics* **28**, 289–292. Characteristics of field turbulent wall pressure fluctuations at large Reynolds numbers.
20. D. M. Chase 1987 *Journal of Sound and Vibration* **112**, 125–147. The character of the turbulent wall pressure spectrum at subconvective wavenumbers and a suggested comprehensive model.
21. A. E. Landmann 1995 *Paper presented at the NASA Langley Workshop on Interior Noise, Hampton VA*, 25–27 *April*. Spatial-temporal boundary layer models for aircraft interior noise.
22. W. R. Graham 1995 *American Institute of Aeronautics and Astronautics Paper* 95-097. A comparison of models for the wavenumber–frequency spectrum of turbulent boundary layer pressures.